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## Aperture Effect Influence and Analysis of Wideband Phased Array Radar

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### Abstract

Wideband phased array becomes a research focus with the development of modern radar technology. Wideband phased array radar is one of the advanced radar technology. However, the aperture fill time results in the beam pointing offset, limits the instantaneous bandwidth and effects of the 1-D target range profile. This paper analyses the scheme of sub-array compensation for aperture fill time. For LFM signals system, we analyze the compensation using filters for aperture fill time, and for stepping frequency signal system, we analyze the scheme of interveinal phase matching, then simulate by Matlab. The simulation results show that the sub array division technology can compensate the aperture effect and increase the instantaneous bandwidth effectively. Both the filter and the interveinal phase matching methods can solve the main lobe expansion and targets range migration; lastly get the high resolution range profile.

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*Keyword:* Wideband phased array; Aperture effect; Sub-array division; Filter; Interveinal phase matching

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### 1. Introduction

With the development of modern radar technology, wideband phased array becomes a research focus. However, when phased array radar is scanning in the wideband and wide-angle range, the aperture fill time makes the signal instantaneous bandwidth limited and beam pointing offset. Moreover, the antenna aperture effect will make the range profile of target defocusing and result in main lobe expansion and

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movement of range profile. Thus appropriate method must be taken to make antenna beam point to the accurate direction, improve the signal instantaneous bandwidth and obtain high-resolution range profile. Considering the development cost and volume of the system, sub-array division is commonly used to compensate the aperture effect. Firstly the paper analyzes the beam pointing offset of wideband phased array caused by aperture effect and gives the method of sub-array division to compensate the aperture effect, and then respectively analyzes the influence caused by aperture effect on range profiles of LFM signal and step-frequency signal wideband phased array, lastly introduces filter compensation technique and interveinal phase matching method.

## 2. Compensation methods for aperture effect of wideband phased array

### 2.1. Aperture effect

Phased array antenna consists of thousands of radiation array elements, which control the beam to scan in certain airspace. Suppose the distance of phased array element is  $d$ , the elements number is  $N$ , linear array aperture is  $L = (N-1)d$ , centre frequency of the radar signal is  $f_0$ , the corresponding wave length  $\lambda_0 = c/f_0$ , instantaneous bandwidth of the radar signal is  $\Delta f$ , so  $\lambda = c/(f_0 + \Delta f)$  is the wavelength corresponding to instantaneous frequency. The speed of light is  $c$ ,  $\theta_0$  is the maximum of antenna beam pointing,  $\theta$  is the angle of target.

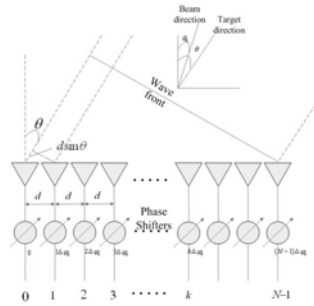


Fig1. The schematic graph of phased array antenna

When antenna pattern of all array elements are isotropic, through analysis and calculation of the phased array, we can get the function of the phased array antenna as:

$$F(\theta) = \sum_{n=0}^{N-1} \exp[jn(\frac{2\pi}{\lambda} d \sin \theta - \Delta \varphi_0)] \quad (1)$$

Where  $\Delta \varphi_0 = (2\pi/\lambda_0)d \sin \theta_0$ . By transforming (1) we can get the normalized amplitude of antenna:

$$|F(\theta)| = \left| \sin \left\{ \frac{N\pi}{c} d [(f_0 + \Delta f) \sin \theta - f_0 \sin \theta_0] \right\} / N \sin \left\{ \frac{\pi}{c} d [(f_0 + \Delta f) \sin \theta - f_0 \sin \theta_0] \right\} \right| \quad (2)$$

According to the antenna maximal beam pointing, we can get the beam pointing offset is defined as:

$$\Delta \theta_f = \theta - \theta_0 = \arcsin[\sin \theta_0 / (1 + \Delta f / f_0)] - \theta_0 \quad (3)$$

Signal frequency changing from  $f_0$  to  $(f_0 + \Delta f)$  results in the antenna beam pointing offset of  $\Delta\theta_f$ . This phenomenon shows that the beam pointing swings in pace with the change of instantaneous frequency, which is known as dispersion phenomenon of beam pointing or aperture effect in phased array antenna.

## 2.2. Sub array division

Compensate the aperture effect to make the beam point to the maximum direction, which can be realized by attaching real-time delay line to each array element. The length of real-time delay lines inserted into array elements  $n$  is  $l_n = nl/(N-1)$ ,  $n = 0, 1, 2, \dots, N-1$ .

Where  $l = L \sin \theta_0 = (N-1)d \sin \theta_0$ . The phase difference between the first and last array element  $\varphi$  can be expressed by:

$$\varphi = 2\pi f_0 [(N-1)d \sin \theta_0 - l] / c \quad (4)$$

When the working frequency changes from  $f_0$  to  $f_0 + \Delta f$ , the expression of maximum beam pointing changing from  $\theta_0$  to  $\theta_1$  can be drawn as:

$$\theta_1 = \arcsin \left\{ \left[ \sin \theta_0 - l / (N-1)d \right] / 1 + \Delta f / f_0 + l / (N-1)d \right\} \quad (5)$$

However, using the real-time delay lines will increase both the volume and the cost of the equipment. So we can use the sub-array division instead. The sub-array division structure is given below:

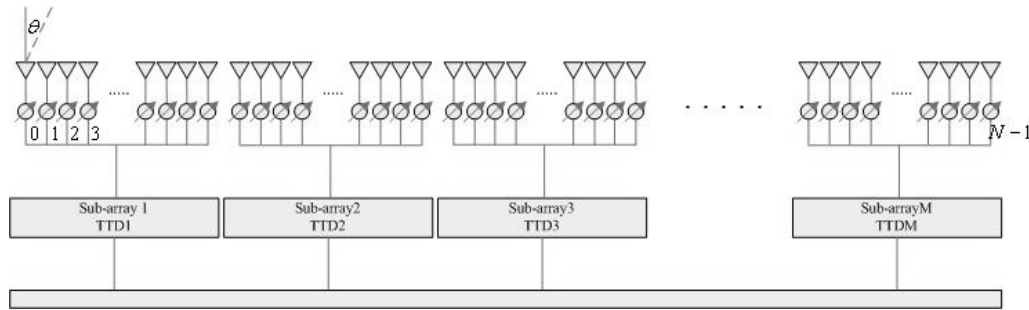


Fig2. The structure graph of subarray division

Phased array antenna will be evenly divided into  $M$  sub-arrays. The length of delay line in sub-array  $m$  is  $l_m = d \sin \theta \cdot mN/M$ ,  $m = 0, 1, 2, \dots, M-1$ . Every sub-array contains  $n_s = N/M$  elements. The phased array antenna is defined as:

$$F(\theta) = \sum_{m=0}^{M-1} F_s(\theta) \exp[j(2\pi mn_s d \sin \theta / \lambda - 2\pi l_m / \lambda)] \quad (6)$$

Where the sub-array is:

$$F_s(\theta) = \sum_{s=0}^{n_s-1} \exp[js(2\pi d \sin \theta / \lambda - \Delta\varphi_0)] \quad (7)$$

From (6) and (7), we can get:

$$|F(\theta)| = \frac{\sin[N\pi d(\sin \theta - \sin \theta_0) / \lambda]}{M \sin[n_s \pi d(\sin \theta - \sin \theta_0) / \lambda]} g \frac{\sin\{n_s \pi d[(f_0 + \Delta f) \sin \theta - f_0 \sin \theta_0] / c\}}{n_s \sin\{\pi d[(f_0 + \Delta f) \sin \theta - f_0 \sin \theta_0] / c\}} \quad (8)$$

The working frequency of phased array antenna changes from  $f_0$  to  $f_0 + \Delta f$  after sub-arrays using the real delay lines. The offset of scanning beam-pointing can be expressed as:

$$\theta_1 = \arcsin\left\{\left[(N-M)/(1+\Delta f/f_0) + N(M-1)\right]\sin\theta_0/(NM-M)\right\} \quad (9)$$

From (9), we consider three specific conditions below:

1. If  $\Delta f = 0$ , then  $\theta_1 = \theta_0$ . The frequency doesn't change and phased array beam-pointing has no offset.
2. If  $M=1$ , then  $\theta_1 = \arcsin[1/(1+\Delta f/f_0) \cdot \sin\theta_0]$ . There is one sub-array. Neither the sub-array division nor real-time delay line is used, so the beam pointing offsets.
3. If  $M=N$ , then  $\theta_1 = \theta_0$ . Because each element has real-time delay line, the beam-pointing has no offset no matter how much the working frequency changes.

### 2.3. Filter

LFM signal is a common wideband signal. The expression of LFM signal can be expressed as:

$$s(t) = \exp[j2\pi(f_0 t + \mu t^2/2)] \quad (10)$$

Where  $f_0$  is the centre frequency of the signal,  $\mu = \Delta f/\tau$  is the rate,  $\tau$  is the pulse bandwidth.

If we choose the first element as the reference zero-phase point, the output of the  $i$ th element is:

$$s_i(t) = s(t) \exp[-j2\pi f_0 i d \sin\theta_0/c + j\pi\mu i^2 (d \sin\theta_0/c)^2 - j2\pi\mu d \sin\theta_0 i t/c] \quad (11)$$

Where  $\exp(-j2\pi f_0 i d \sin\theta_0/c)$  is the phase difference between the  $i$ th and the reference element that can be compensated by phase shifters after the  $i$ th element.  $\exp(j\pi\mu i^2 (d \sin\theta_0/c)^2)$  is far less than 1 so that can be omitted. Thus the synthetic output of phased array as follows:

$$s_o(t) = \sum_{i=0}^{N-1} s_i(t) \exp(j2\pi f_0 i d \sin\theta_0/c) \quad (12)$$

After dechirp processing of echo signals, the output of the signal can be defined as:

$$s_{dechirp}(t) = s_o(t)s^*(t) \quad (13)$$

After FFT it can get 1-D range profile. The aperture fill time will result in main lobe expansion and range profile offset. We can choose the centre element as the reference zero-phase point. Thus, the synthetic output is:

$$s_o(t) = \sum_{i=-(N-1)/2}^{i=N-1/2} s_i(t) \exp(j2\pi f_0 i d \sin\theta_0/c) = s(t) \sin(\pi N \mu d \sin\theta_0 t/c) / \sin(\pi \mu d \sin\theta_0 t/c) \quad (14)$$

The main lobe expansion of range profile is mainly described in amplitude weighting of echo signals.

$$h(t) = \sin(\pi N \mu d \sin\theta_0 t/c) / \sin(\pi \mu d \sin\theta_0 t/c) \quad (15)$$

When the phased array antenna structure and the beam scanning angle are known, the filter  $h(t)$  can be determined. The weighted coefficient of the filter is  $1/h(t)$ , we can compensate the main lobe expansion.

### 2.4. Interveinal phase matching.

Stepping Frequency Signal gets the wideband synthetic signal by emitting pulse train whose frequency is gradually increasing. Assumed that the initial frequency of the stepping frequency signal is  $f_0$ , the stepped frequency size is  $\Delta f$ , the pulse-width is  $T_p$ , the pulse repeat period is  $T_r$ , the number of pulses is  $N$ . So the time-domain expression of the stepping frequency signal is:

$$s(t) = \sum_{n=0}^{N-1} \text{rect}[(t - nT_r) / T_p] \exp[j2\pi(f_0 + n\Delta f)t] \quad (16)$$

Apply when stepping frequency is  $[f_0 + (N-1)\Delta f / 2]$  the beam pointing is  $\theta_0$ . The phase-shift value in each element is defined as:

$$\varphi_m = 2\pi[f_0 + (N-1)\Delta f / 2]md \sin \theta_0 / c \quad (17)$$

When the target range is  $R$  ( $\tau_d = 2R / c$ ), the sample of synthetic echo signal is:

$$x_w(n) = \frac{\sin[\pi M(\frac{N-1}{2} - n)\Delta f \frac{d \sin \theta_0}{c}]}{\sin[\pi(\frac{N-1}{2} - n)\Delta f \frac{d \sin \theta_0}{c}]} \square e^{-j\pi(M-1)(\frac{N-1}{2} - n)\Delta f \frac{d \sin \theta_0}{c}} e^{-j2\pi(f_0 + n\Delta f)(\tau_d + \frac{M-1}{2} \frac{d \sin \theta_0}{c})} \quad (18)$$

After IFFT, it get the range profile as follows:

$$y_w(l) = \frac{1}{N} \sum_{n=0}^{N-1} x_w(n) \exp(j2\pi n / N) \quad (19)$$

As the stepping frequency signal gets the wideband equivalent synthetic signal by emitting a stepping frequency pulse train, of which each pulse is a single-frequency signal. So the interveinal phase matching can be used to compensate the aperture effect.

$$\varphi_{nm} = \varphi = (f_0 + n\Delta f)2\pi[(M-1)/2 - m]d \sin \theta_0 / c \quad \Delta\varphi_{nm} = n\Delta f 2\pi[(M-1)/2 - m]d \sin \theta_0 / c \quad (20)$$

If we change the phase matching value of each array element in stepping frequency sub pulse by(20). After interveinal phase matching with (20), the synthetic sample of echo is:

$$x_b(n) = \exp[-j2\pi\tau_d(f_0 + n\Delta f)] \quad (21)$$

From (23), we can know that the interveinal phase matching can eliminate the aperture effect.

### 3. Experiments and Simulation Analysis

The simulation parameters are defined as Radar centre frequency  $f_0 = 3\text{GHz}$ . The array elements is  $N=128$ . The spacing between arrays is  $d = \lambda_0 / 2 = c / (2f_0)$ . The maximum beam pointing is  $\theta_0 = \pi / 3$ . Instantaneous bandwidth is  $\Delta f = 200\text{MHz}$ . The speed of light  $c = 3e8 \text{ m/s}$ .

Assumed that the number of sub arrays and each has elements. From Fig3, we can see that only using the phase shifters, the beam pointing is  $54.28^\circ$  with a deviation of  $5.72^\circ$ . while using the sub-array structure, the maximum beam pointing is  $59.97^\circ$  with a small deviation of  $0.03^\circ$ . So the sub-array division effectively corrects the beam pointing and compensates the aperture effect.

Suppose that the bandwidth of LFM  $B = 3e8\text{MHz}$ , the pulse width is  $\tau = 5e-6 \text{ s}$  and the LFM rate  $\mu = B / \tau = 300e6 / 5e-6 = 6e13$ . After using filter compensation, we can see that the compensation for the peak migration of 1-D pulse compressed range profile and main lobe migration is effective from Fig4. Besides, both the resolution and accuracy of the range are improved. All can show that filters can compensate the range profile migration and main lobe expansion caused by aperture effect.

Suppose that the stepped frequency is  $\Delta f = 1\text{MHz}$ , the number of pulse  $M = 1000$ , closely located point targets  $R = [50 \ 52 \ 56] \text{m}$ , the antenna aperture  $L = 10\text{m}$ . According to the parameters, the range resolution  $\delta r = c / (2N\Delta f) = 0.15\text{m}$ . In theory it can distinguish three targets fully, but in practice we can't make it for aperture effect results in decline of range resolution and main lobe expansion. After using the method of interveinal phase matching, we compensate the phase shift caused by aperture fill time to

distinguish the peak of three point targets clearly and correct the 1-D range profile. The signal noise ratio is improved as well.

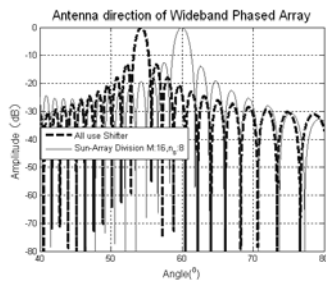


Fig3. Beam pointing graph of sub-array division.

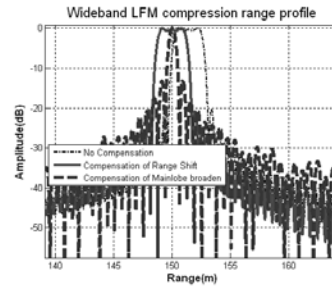
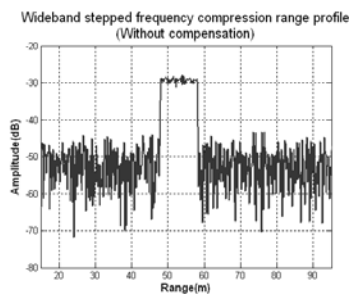
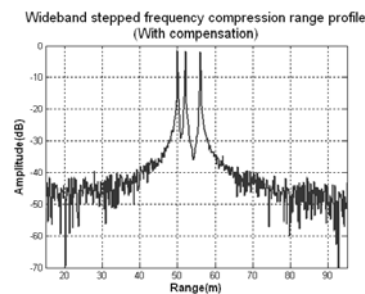


Fig4. Effect Graph of filter compensation



(a) Non-compensation pulse compression



(b) interval phase matching pulse compression

Fig5. The compensation result of interval phase matching

## 4. Conclusions

Using the classical sub-array division method we can make beam point to default maximum direction and compensate aperture effect effectively. Filters are used to compensate main lobe expansion and targets range migration when LFM signal is used. Similarly, we use the method of interval phase matching to compensate the aperture effect when stepping frequency signal is used.

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